

Optical Waveguide Theory (D)



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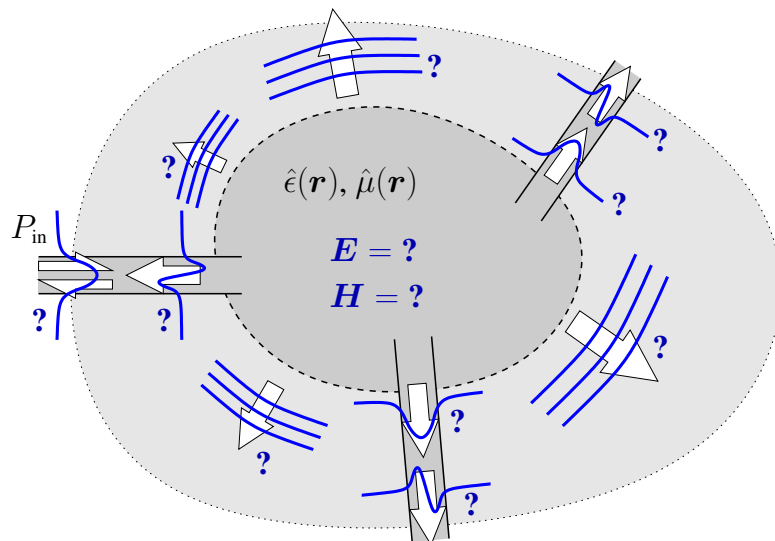
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Course overview

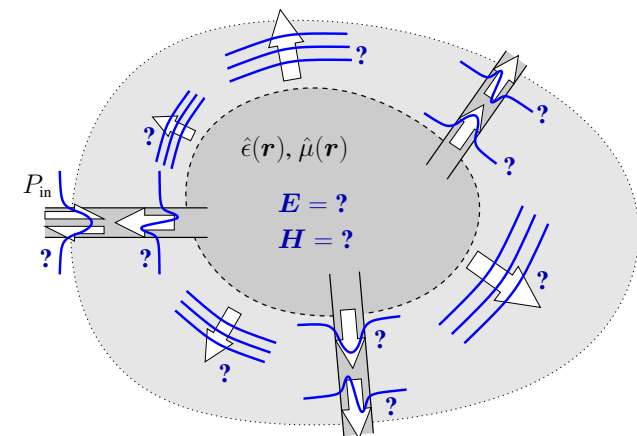
Optical waveguide theory

- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D **Classes of simulation tasks: scattering problems, mode analysis, resonance problems.**
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
 - Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
 - Oblique semi-guided waves: 2-D integrated optics.
 - Summary, concluding remarks.

Guided wave scattering problems, schematically



Guided wave scattering problems, schematically

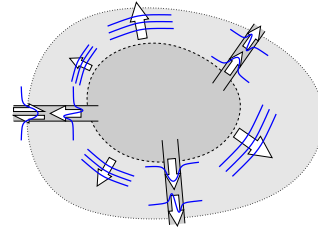


Given $\hat{\epsilon}(\mathbf{r}), \hat{\mu}(\mathbf{r})$ & external excitation (incoming guided mode),
determine \mathbf{E}, \mathbf{H} within the computational domain
& determine the optical power carried by outgoing waves.

Scattering problems, time domain

(TD)

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t), \mathbf{H}(\mathbf{r}, t), \\ \nabla \times \mathbf{E} = -\mu_0 \hat{\mu} \dot{\mathbf{H}}, \\ \nabla \times \mathbf{H} = \epsilon_0 \hat{\epsilon} \dot{\mathbf{E}}. \end{aligned}$$



- $\begin{pmatrix} 3\text{-D} \\ 2\text{-D} \\ 1\text{-D} \end{pmatrix}$ computational domain \times time interval.
- Initial & boundary conditions \longleftrightarrow incident waves.
- “Local” time-explicit iterative schemes possible (e.g. FDTD).
- Time evolution available; direct modeling of pulse propagation.
- Dispersion (...?).
- Guided wave excitation (...?).
- Fourier transform \longrightarrow spectral information.

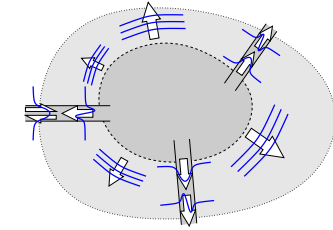
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Scattering problems, frequency domain

(FD)

$$\begin{aligned} \mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}), \sim \exp(i\omega t), \\ \nabla \times \mathbf{E} = -i\omega\mu_0 \hat{\mu} \mathbf{H}, \\ \nabla \times \mathbf{H} = i\omega\epsilon_0 \hat{\epsilon} \mathbf{E}. \end{aligned}$$



- $\begin{pmatrix} 3\text{-D} \\ 2\text{-D} \\ 1\text{-D} \end{pmatrix}$ computational domain.
- “ $\overline{\mathbf{M}(\text{field})} = \overline{(\text{excitation})}$ ”; matrix needs to be determined, stored; system needs to be solved.
- Spectral information directly available.
- Dispersion — straightforward.
- Guided wave excitation — straightforward.
- Fourier transform \longrightarrow time evolution / pulse propagation.

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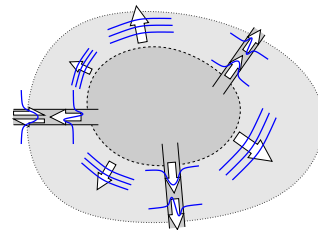
Open problems

(TD & FD)

“Open” spatial computational domain

~> boundary conditions need to

- permit outgoing radiated fields & outgoing (reflected) guided modes to exit the domain,
- launch the incoming external excitation.
- ~> simulate a nonexistent boundary, an unlimited domain.



- Keywords:
- transparent-influx boundary conditions,
 - absorbing boundary conditions,
 - perfectly matched layers (PMLs).



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2-D problems

$$\hat{\epsilon} = \epsilon \hat{1}, \quad \hat{\mu} = \mu \hat{1}, \quad \sim \exp(i\omega t) \quad (\text{FD})$$

Assume $\partial_y \epsilon = 0, \partial_y \mu = 0$; consider solutions $\partial_y \mathbf{E} = 0, \partial_y \mathbf{H} = 0$:

$$\begin{pmatrix} -\partial_z E_y \\ \partial_z E_x - \partial_x E_z \\ \partial_x E_y \end{pmatrix} = -i\omega\mu_0\mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} -\partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y \end{pmatrix} = i\omega\epsilon_0\epsilon \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$



Two decoupled sets of equations:

- $\{E_y, H_x, H_z\}$: transverse electric (TE) fields, $\mathbf{E} \perp x\text{-}z\text{-plane}$.
- $\{H_y, E_x, E_z\}$: transverse magnetic (TM) fields, $\mathbf{H} \perp x\text{-}z\text{-plane}$.

(Different conventions on the use of TE, TM.)

(Applies also to the TD.)

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2-D TE waves

$$k^2 = \omega^2/c^2 = \omega^2 \epsilon_0 \mu_0 \quad (\text{FD})$$

- Principal component E_y ,

$$H_x = \frac{-i}{\omega \mu_0 \mu} \partial_z E_y, \quad H_z = \frac{i}{\omega \mu_0 \mu} \partial_x E_y, \quad i\omega \epsilon_0 \epsilon E_y = \partial_z H_x - \partial_x H_z$$

$$\hookrightarrow \partial_x \frac{1}{\mu} \partial_x E_y + \partial_z \frac{1}{\mu} \partial_z E_y + k^2 \epsilon E_y = 0. \quad (*)$$

- Continuity of E_y , $\frac{1}{\mu} \partial_n E_y$ required at interfaces with normal \mathbf{n} .

- If $\mu = 1$: $\epsilon(x, z)$ (!)

$$\hookrightarrow \partial_x^2 E_y + \partial_z^2 E_y + k^2 \epsilon E_y = 0, \quad (**)$$

scalar 2-D (TE) Helmholtz equation (E_y , $\partial_n E_y$ continuous).

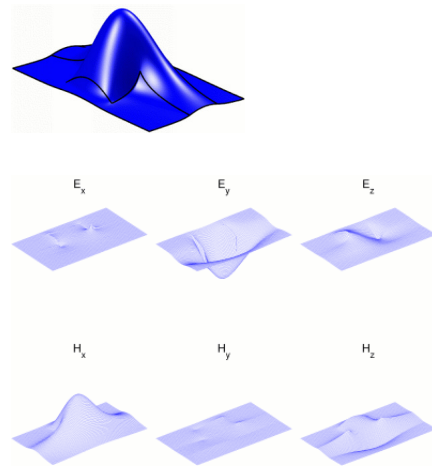
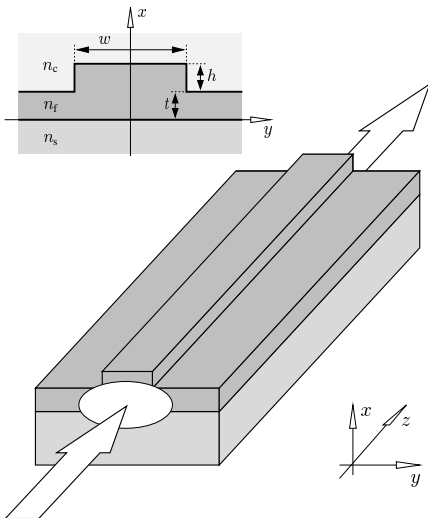
(Reflection / transmission problems: s-polarized waves satisfy (*), (**).)



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Rib waveguide

... variant of an integrated optical waveguide with 2-D confinement



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2-D TM waves

$$k^2 = \omega^2/c^2 = \omega^2 \epsilon_0 \mu_0 \quad (\text{FD})$$

- Principal component H_y ,

$$E_x = \frac{i}{\omega \epsilon_0 \epsilon} \partial_z H_y, \quad E_z = \frac{-i}{\omega \epsilon_0 \epsilon} \partial_x H_y, \quad -i\omega \mu_0 \mu H_y = \partial_z E_x - \partial_x E_z$$

$$\hookrightarrow \partial_x \frac{1}{\epsilon} \partial_x H_y + \partial_z \frac{1}{\epsilon} \partial_z H_y + k^2 \mu H_y = 0. \quad (*)$$

- Continuity of H_y , $\frac{1}{\epsilon} \partial_n H_y$ required at interfaces with normal \mathbf{n} .

- If $\mu = 1$: $\epsilon(x, z)$ (!)

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scalar 2-D (TM) Helmholtz equation (H_y , $\frac{1}{\epsilon} \partial_n H_y$ continuous).

(Reflection / transmission problems: p-polarized waves satisfy (*), (**).)



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Waveguides: Mode problems

$$\nabla \times \mathbf{E} = -i\omega \mu_0 \mu \mathbf{H}, \quad \nabla \times \mathbf{H} = i\omega \epsilon_0 \epsilon \mathbf{E}. \quad \sim \exp(i\omega t) \quad (\text{FD})$$

- Waveguide: a system that is homogeneous along its axis z ,
 $\partial_z \epsilon = 0$, $\partial_z \mu = 0$, $\partial_z n = 0$.

- Look for solutions (modes) that vary harmonically with z :

$$\mathbf{E}(x, y, z) = \bar{\mathbf{E}}(x, y) e^{-i\beta z}, \quad \mathbf{H}(x, y, z) = \bar{\mathbf{H}}(x, y) e^{-i\beta z},$$

mode profile $\bar{\mathbf{E}}, \bar{\mathbf{H}}$, propagation constant β .

(drop \neg)

$$\begin{pmatrix} \partial_y E_z + i\beta E_y \\ -i\beta E_x - \partial_x E_z \\ \partial_x E_y - \partial_y E_x \end{pmatrix} = -i\omega \mu_0 \mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} \partial_y H_z + i\beta H_y \\ -i\beta H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix} = i\omega \epsilon_0 \epsilon \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix},$$

vectorial mode equations, variants.

(...)



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Waveguides: Mode equations

- Where $\epsilon(\mathbf{r})$, $\mu(\mathbf{r})$: $\sim \exp(i\omega t)$ (FD)

$$\Delta \tilde{\mathbf{E}} + k^2 \epsilon \mu \tilde{\mathbf{E}} = 0, \quad \Delta \tilde{\mathbf{H}} + k^2 \epsilon \mu \tilde{\mathbf{H}} = 0$$

$$\begin{aligned} \partial_x^2 \mathbf{E} + \partial_y^2 \mathbf{E} + (k^2 \epsilon \mu - \beta^2) \mathbf{E} &= 0, \\ \partial_x^2 \mathbf{H} + \partial_y^2 \mathbf{H} + (k^2 \epsilon \mu - \beta^2) \mathbf{H} &= 0, \end{aligned}$$

scalar **mode equation**, valid for all components of \mathbf{E} , \mathbf{H} ,
to be supplemented by suitable **boundary** and **interface conditions**.

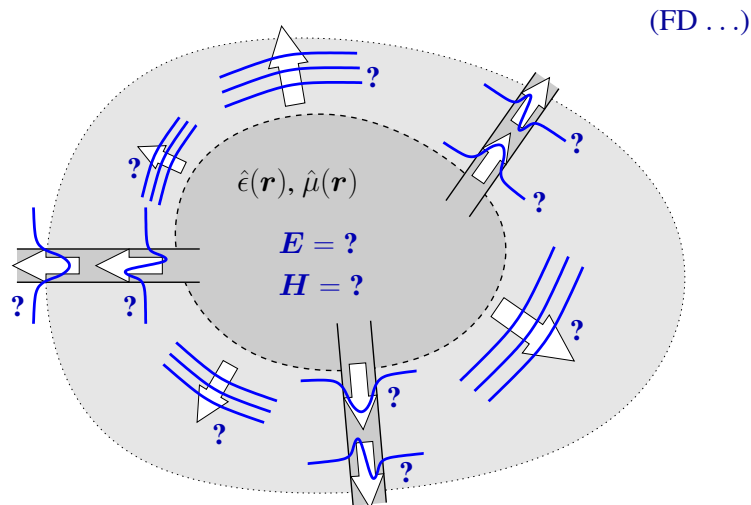
- ↔ **Eigenvalue problem** with eigenvalue β , eigenfunction \mathbf{E} , \mathbf{H} ,
“ $\mathbf{M}(\beta)$ (profile) = 0”.

- **Guided modes**: discrete $\beta \in \mathbb{R}$, $\iint S_z dx dz < \infty$.
- **Radiation modes**: continuum of $\beta^2 \in \mathbb{R}$, oscillating external fields.
- **Leaky modes**: discrete $\beta \in \mathbb{C}$, outgoing wave boundary conditions.
- (...)

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Resonance problems

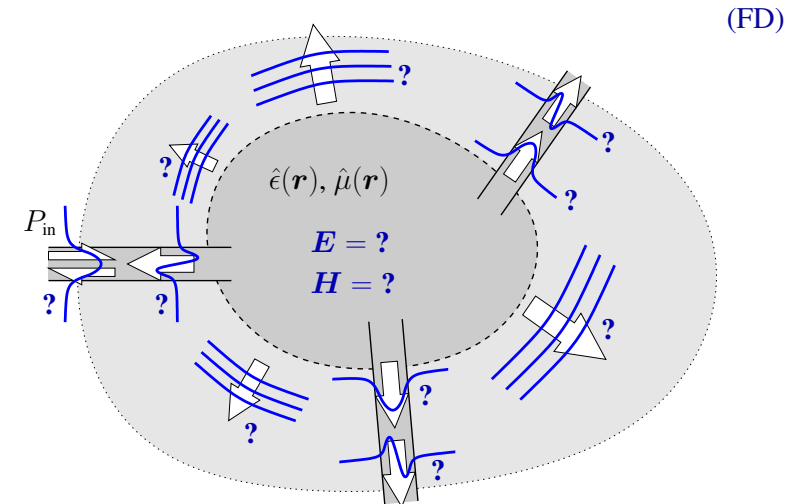


Omit excitation, look for nonzero solutions that decay in time.

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Guided wave scattering problems



Given external excitation $\sim \exp(i\omega t)$, $\omega \in \mathbb{R}$.

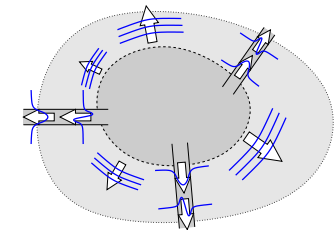
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Resonance problems

(FD ...)

$\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}), \sim \exp(i\omega t)$, $\omega = ?$
 $\nabla \times \mathbf{E} = -i\omega \mu_0 \hat{\mu} \mathbf{H}$,
 $\nabla \times \mathbf{H} = i\omega \epsilon_0 \hat{\epsilon} \mathbf{E}$,
 & outgoing wave boundary conditions.



- Look for nonzero solutions with $\omega \in \mathbb{C}$ that oscillate and decay (slowly ...) in time.
- “ $\mathbf{M}(\omega)$ (field) = 0”, **eigenvalue problem**.
- Solutions: discrete eigenfrequencies ω , resonant mode profiles.

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Scalar approximation

Linear, isotropic, nonmagnetic media, $\epsilon = n^2$;
 a structure with “small” variations in ϵ :

A scalar approximation may be adequate,

$$\nabla \cdot (\epsilon \mathbf{E}) \approx \epsilon \nabla \cdot \mathbf{E}$$

$$\Delta \psi - \frac{1}{c^2} \epsilon \ddot{\psi} = 0, \quad (\text{TD})$$

$$\Delta \psi + k^2 \epsilon \psi = 0, \quad (\text{FD})$$

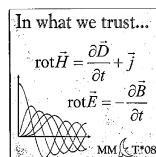
satisfied by all components ψ of \mathbf{E} , \mathbf{H} .

(Applicable to basically all types of problems.)

Upcoming

Next lectures:

- Normal modes of dielectric optical waveguides, mode interference.
- Examples for dielectric optical waveguides.
- Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.



Beam propagation method

- Starting point: $\Delta \psi + k^2 \epsilon \psi = 0$, $\sim \exp(i\omega t)$ (FD)
 “small” changes in $\epsilon = n^2$ along a propagation coordinate z .

- Ansatz: $\psi(x, y, z) = \psi_0(x, y, z) e^{-ikn_r z}$,
 reference effective index n_r ,
 assume that ψ_0 varies “slowly” along z \leftrightarrow neglect $\partial_z^2 \psi_0$.

$$\leftarrow -i2kn_r \partial_z \psi_0 + (\partial_x^2 + \partial_y^2) \psi_0 + k^2 (\epsilon - n_r^2) \psi_0 = 0,$$

PDE of first order in z , solved as an initial value problem.

- Restriction to unidirectional propagation, reflections are neglected.
- Paraxial propagation, errors for waves with effective indices $\neq n_r$.

(Many variants (vectorial, wide-angle, bi-directional, ...) have been proposed.)

(Other ways of motivating the approximation exist.)

(Term “BPM” in use also for other types of methods.)

- Keywords: Paraxial approximation,
 Slowly-varying-envelope approximation (SVEA),
 Beam propagation method (BPM).