

Optical Waveguide Theory (G)



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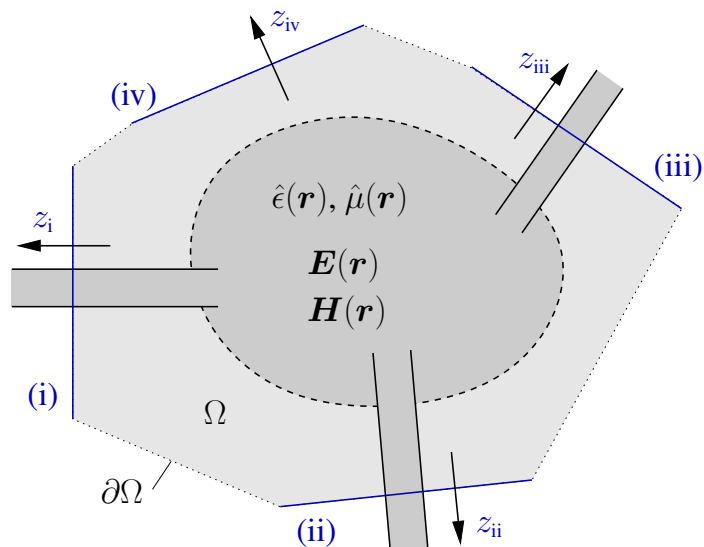
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Course overview

Optical waveguide theory

- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
 - Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
 - Oblique semi-guided waves: 2-D integrated optics.
 - Summary, concluding remarks.

PICs, OICs, scattering matrices



Scattering matrices, prerequisites

- Passive, linear circuit. $\sim \exp(i\omega t)$ (FD)
- (Computational) domain of interest Ω , its boundary $\partial\Omega$.
- Connecting channels: lossless waveguides (or “half-spaces”).
- Physical ports $p = i, ii, \dots$: waveguide cross-section planes, local coordinates x_p, y_p, z_p ; local axis z_p oriented outwards of Ω .
- Establish sets \mathcal{N}_p of propagating directional normal modes $\{\psi_{p,m}^d := (\mathbf{E}_{p,m}^d, \mathbf{H}_{p,m}^d), \beta_{p,m}; d = f, b\}$ on each port p .
(Restriction to propagating fields: a condition on port positioning / a model assumption.)
- Ports & modes are such that all mode fields vanish on all “other” port planes, and on $\partial\Omega$ outside the ports.

Field on port plane p and “outside”:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x_p, y_p, z_p) = \sum_{m \in \mathcal{N}_p} F_{p,m} \psi_{p,m}^f(x_p, y_p) e^{-i\beta_{p,m}z_p} + B_{p,m} \psi_{p,m}^b(x_p, y_p) e^{i\beta_{p,m}z_p}$$

Scattering matrices

- Merge all mode indices $\{m\}$ and port IDs $\{p\}$ into one set of mode identifiers $\{\nu\}$, $\mathcal{N} = \cup_p \mathcal{N}_p$. $\sim \exp(i\omega t)$ (FD)
- Assert that $\psi_{p,\cdot}(\mathbf{r}) = 0$ for all $\mathbf{r} \in \partial\Omega$, $\mathbf{r} \notin \text{port } p$.
- Field on $\partial\Omega$: $\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \sum_{\nu \in \mathcal{N}} \{F_\nu \psi_\nu^f + B_\nu \psi_\nu^b\}$. (Position arguments omitted.)
- B_ν : \sim incident modes, traveling towards the interior of Ω .
 F_ν : \sim outgoing modes, traveling towards the exterior of Ω .
 Combine into amplitude vectors \mathbf{B}, \mathbf{F} .

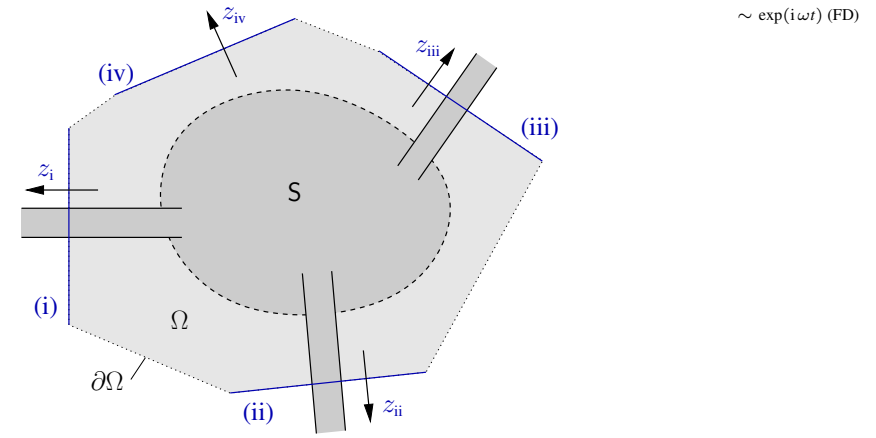
Linear circuit \longleftrightarrow linear dependence of \mathbf{F} on \mathbf{B} ,
 Scattering matrix \mathbf{S} of the circuit: $\mathbf{F} = \mathbf{S}\mathbf{B}$, $\mathbf{S} = (S_{\nu\mu})$.

- $S_{\nu\nu}$: $\sim (\nu, b) \rightarrow (\nu, f)$, reflection coefficient for mode ν .
- $S_{\nu\mu}$: $\sim (\mu, b) \rightarrow (\nu, f)$, transmission coefficient for modes μ, ν .

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PICs, OICs, scattering matrices, scenarios

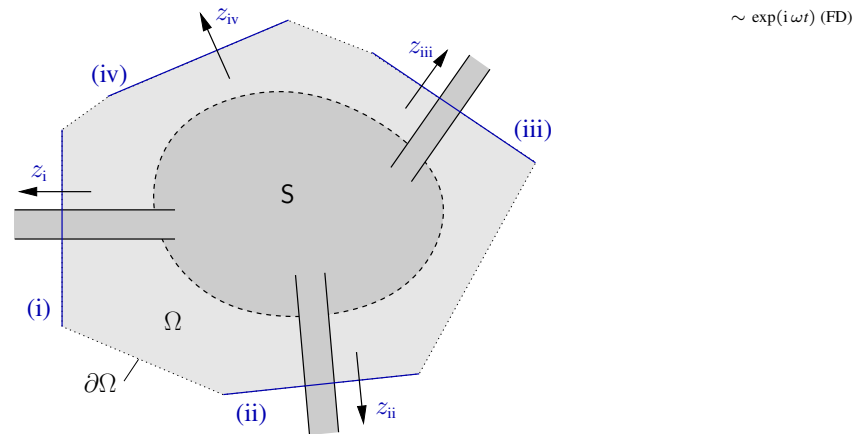


- Scenario: Full matrix \mathbf{S} , including guided and radiation modes, large $\dim \mathbf{S} \leftrightarrow$ theoretical results.

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PICs, OICs, scattering matrices, scenarios



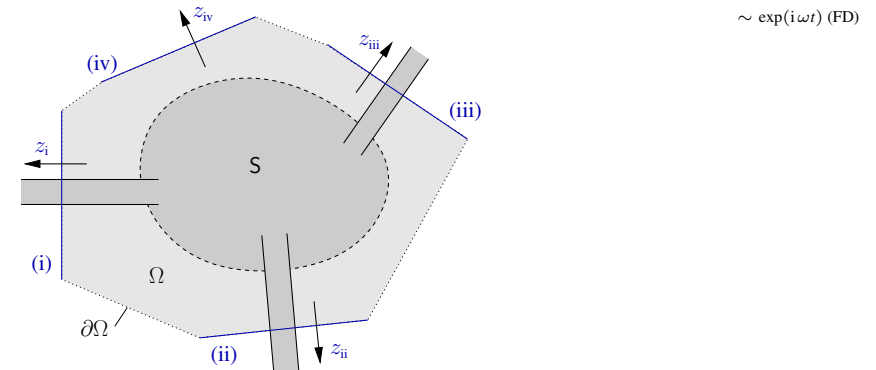
- Scenario: Restrict to a specific set of (guided) modes, or: Only a small set of guided modes are relevant: small $\dim \mathbf{S} = N \times N \leftrightarrow$ an N -port circuit, a $2N$ -pole.

(N : the total number of relevant modes, not the number of ports.)

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Scattering matrices, port plane positions



- Shift port plane of mode ν by Δz_ν : $F_\nu \rightarrow F'_\nu = F_\nu e^{-i\beta_\nu \Delta z_\nu}$,
 Shift port plane of mode μ by Δz_μ : $B_\mu \rightarrow B'_\mu = B_\mu e^{i\beta_\mu \Delta z_\mu}$,
 $\curvearrowright F'_\nu = S'_{\nu\mu} B'_\mu$, $S'_{\nu\mu} = S_{\nu\mu} e^{-i(\beta_\nu \Delta z_\nu + \beta_\mu \Delta z_\mu)}$.

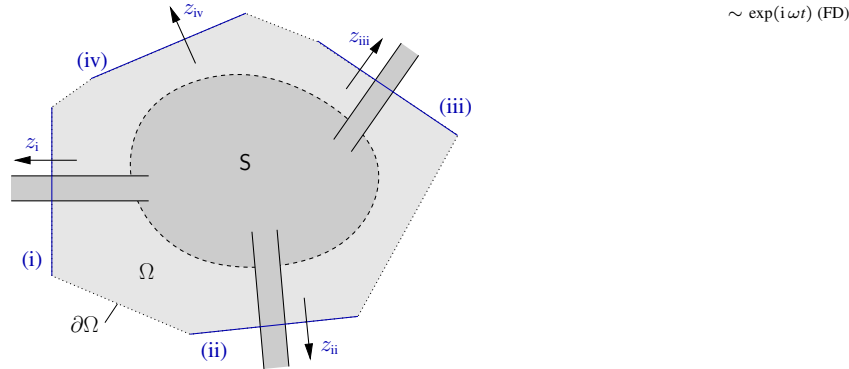
(Moving port planes \leftrightarrow Phase change in reflection/transmission coefficients.)

(Moving port planes \leftrightarrow No effect on reflectances/transmittances.)

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Scattering matrices, port mode orthogonality



$\sim \exp(i\omega t)$ (FD)

- Orthogonality relations on port plane p :

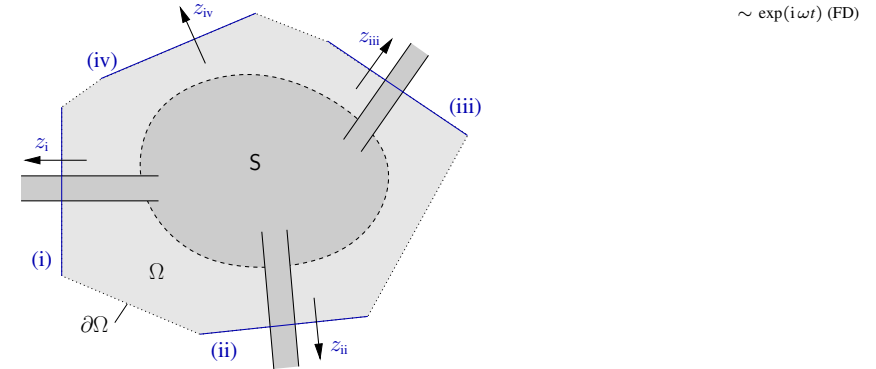
$$(\mathbf{E}_a, \mathbf{H}_a; \mathbf{E}_b, \mathbf{H}_b) = \frac{1}{4} \iint_p (\mathbf{E}_a^* \mathbf{H}_b - \mathbf{E}_a \mathbf{H}_b^* + \mathbf{H}_a^* \mathbf{E}_b - \mathbf{H}_a \mathbf{E}_b^*) dx_p dy_p$$

$$(\psi_{p,i}^d; \psi_{p,m}^r) = \pm \delta_{dr} \delta_{lm} P_{p,m} \quad (\text{Things restricted to propagating modes.})$$

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Scattering matrices, port mode orthogonality



$\sim \exp(i\omega t)$ (FD)

- Extend to the full boundary $\partial\Omega$:

$$(\mathbf{E}_a, \mathbf{H}_a; \mathbf{E}_b, \mathbf{H}_b) := \frac{1}{4} \int_{\partial\Omega} (\mathbf{E}_a^* \times \mathbf{H}_b + \mathbf{E}_b \times \mathbf{H}_a^*) \cdot d\mathbf{a}$$

$$\curvearrowright (\psi_{p,i}^d; \psi_{q,m}^r) = \pm \delta_{dr} \delta_{pq} \delta_{lm} P_{p,m} \quad \text{or} \quad (\psi_{\nu}^d; \psi_{\mu}^r) = \pm \delta_{dr} \delta_{\nu\mu} P_{\nu}.$$

(Modes belonging to different ports are mutually orthogonal.)

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Scattering matrices, power balance



$\sim \exp(i\omega t)$ (FD)

- Net power outflow across the border of the circuit:

$$P = \int_{\partial\Omega} \mathbf{S} \cdot d\mathbf{a} = (\mathbf{E}, \mathbf{H}; \mathbf{E}, \mathbf{H}) = \sum_p \sum_{m \in \mathcal{N}_p} (|F_{p,m}|^2 - |B_{p,m}|^2) P_{p,m}$$

$$= \sum_{\nu \in \mathcal{N}} (|F_{\nu}|^2 - |B_{\nu}|^2) P_{\nu},$$

$$|B_{\mu}|^2 P_{\mu}: \text{ incident power carried by mode } \mu,$$

$$|F_{\nu}|^2 P_{\nu}: \text{ outgoing power carried by mode } \nu, \quad F_{\nu} = S_{\nu\mu} B_{\mu}.$$

$$|S_{\nu\mu}|^2 \frac{P_{\nu}}{P_{\mu}} = \frac{|F_{\nu}|^2 P_{\nu}}{|B_{\mu}|^2 P_{\mu}}, \quad \mu \neq \nu: \text{ power transmittance } \mu \rightarrow \nu,$$

$$\mu = \nu: \text{ power reflectance for mode } \nu.$$

(Uniform normalized modes, $P_{\nu} = P_{\mu}$: transmittances are directly given by elements of the scattering matrix.)

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Scattering matrices, power balance



$\sim \exp(i\omega t)$ (FD)

- Net power outflow across the border of the circuit:

$$P = \int_{\partial\Omega} \mathbf{S} \cdot d\mathbf{a} = (\mathbf{E}, \mathbf{H}; \mathbf{E}, \mathbf{H}) = P_0 (\mathbf{B}^* \cdot (\mathbf{S}^{\dagger} \mathbf{S} - \mathbf{1}) \mathbf{B}),$$

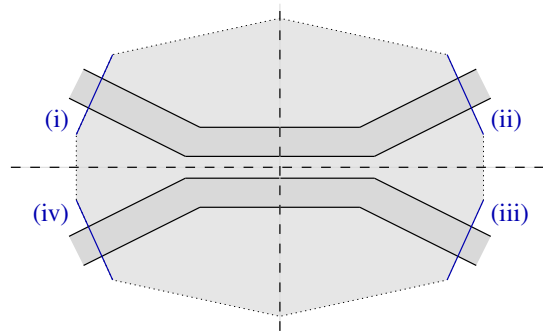
uniform normalization, $P_{\nu} = P_0$ for all ν .

- Lossless circuit $\iff \int_{\partial\Omega} \mathbf{S} \cdot d\mathbf{a} = 0 \iff \mathbf{S}^{\dagger} \mathbf{S} = \mathbf{1}$,
the scattering matrix of a lossless circuit is unitary.

- Lossy circuit $\iff \int_{\partial\Omega} \mathbf{S} \cdot d\mathbf{a} \leq 0 \iff \mathbf{B}^* \cdot \mathbf{S}^{\dagger} \mathbf{S} \mathbf{B} \leq \mathbf{B}^* \mathbf{B}$,
 $\sum_{\nu} |S_{\nu\mu}|^2 \leq 1$ for all μ . (The sum of transmittances mode μ to all other modes ν is less than one.)
(Interior lossy media, or radiative losses: outgoing propagating modes not taken into account.)

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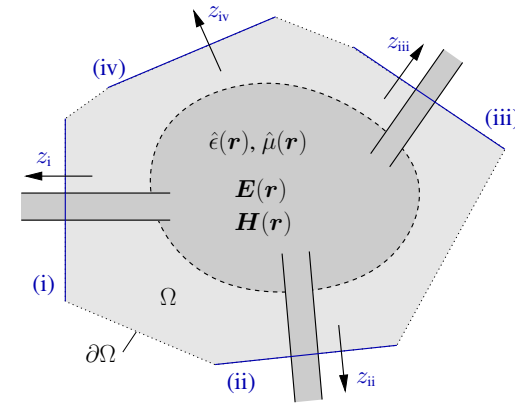
Scattering matrices, symmetry



Circuit with specific spatial symmetry
& symmetrical setting of the port planes

↪ respective symmetry in related coefficients of \mathbf{S} ,
symmetric power transmission properties.

Scattering matrices, reciprocity



Circuit properties
for reversed
wave propagation?
 $S_{\nu\mu} \longleftrightarrow S_{\mu\nu}$?

- $\mathbf{E}_1, \mathbf{H}_1$ and $\mathbf{E}_2, \mathbf{H}_2$ solve $\nabla \times \mathbf{E} = -i\omega\mu_0\hat{\mu}\mathbf{H}$, $\nabla \times \mathbf{H} = i\omega\epsilon_0\hat{\epsilon}\mathbf{E}$.
- ↪ $\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{H}_1 \times \mathbf{E}_2) = 0$, if $\hat{\epsilon}$ and $\hat{\mu}$ are symmetric.
(i.e. if $\hat{\epsilon}^T = \epsilon$, $\hat{\mu}^T = \mu$.)
(Note: order of factors, no complex conjugates.)

Scattering matrices, reciprocity

- $\mathbf{E}_1, \mathbf{H}_1$ and $\mathbf{E}_2, \mathbf{H}_2$ solve $\nabla \times \mathbf{E} = -i\omega\mu_0\hat{\mu}\mathbf{H}$, $\nabla \times \mathbf{H} = i\omega\epsilon_0\hat{\epsilon}\mathbf{E}$

↪ $\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{H}_1 \times \mathbf{E}_2) = 0$, if $\hat{\epsilon}$ and $\hat{\mu}$ are symmetric,

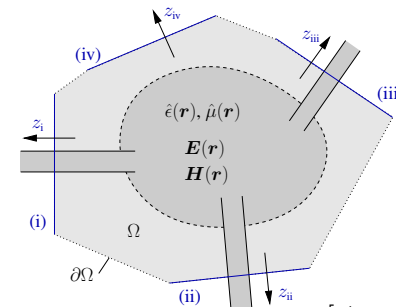
↪
$$0 = \int_{\Omega} \nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{H}_1 \times \mathbf{E}_2) d^3r = \int_{\partial\Omega} (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{H}_1 \times \mathbf{E}_2) \cdot d\mathbf{a}.$$

- Fields on $\partial\Omega$: $\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_j = \sum_{\nu \in \mathcal{N}} \{F_{j,\nu}\psi_{\nu}^f + B_{j,\nu}\psi_{\nu}^b\}$, $j = 1, 2$,

$$[\psi_a; \psi_b] := \int_{\partial\Omega} (\mathbf{E}_a \times \mathbf{H}_b + \mathbf{H}_a \times \mathbf{E}_b) \cdot d\mathbf{a},$$

↪
$$0 = \sum_{\nu} \sum_{\mu} \left(F_{1,\nu}F_{2,\mu}[\psi_{\nu}^f; \psi_{\mu}^f] + F_{1,\nu}B_{2,\mu}[\psi_{\nu}^f; \psi_{\mu}^b] + B_{1,\nu}F_{2,\mu}[\psi_{\nu}^b; \psi_{\mu}^f] + B_{1,\nu}B_{2,\mu}[\psi_{\nu}^b; \psi_{\mu}^b] \right).$$

Scattering matrices, reciprocity



$$[\psi_a; \psi_b] := \int_{\partial\Omega} (\mathbf{E}_a \times \mathbf{H}_b + \mathbf{H}_a \times \mathbf{E}_b) \cdot d\mathbf{a}.$$

- $[\psi_{\nu}; \psi_{\mu}] = 0$, if ν and μ relate to different ports.
- If ν and μ relate to the same port plane p :
$$[\psi_{\nu}^r; \psi_{\mu}^d] = \iint_p (E_{\nu x}^r H_{\mu y}^d - E_{\nu y}^r H_{\mu x}^d - H_{\nu y}^r E_{\mu x}^d + H_{\nu x}^r E_{\mu y}^d) dx_p dy_p.$$

Scattering matrices, reciprocity

- If ν and μ relate to the same port plane p :

$$[\psi_\nu^r; \psi_\mu^d] = \iint_p (E_{\nu x}^r H_{\mu y}^d - E_{\nu y}^r H_{\mu x}^d - H_{\nu y}^r E_{\mu x}^d + H_{\nu x}^r E_{\mu y}^d) dx_p dy_p.$$

- Compare with the modal orthogonality relations on port plane p , for propagating modes with real transverse components:

$$(\psi_\nu^r; \psi_\mu^d) = \frac{1}{4} \iint_p (E_{\nu x}^r H_{\mu y}^d - E_{\nu y}^r H_{\mu x}^d + H_{\nu y}^r E_{\mu x}^d - H_{\nu x}^r E_{\mu y}^d) dx_p dy_p,$$

$$(\psi_\nu^f; \psi_\mu^f) = \delta_{\nu\mu} P_\nu, \quad (\psi_\nu^b; \psi_\mu^b) = -\delta_{\nu\mu} P_\nu, \quad (\psi_\nu^f; \psi_\mu^b) = (\psi_\nu^b; \psi_\mu^f) = 0.$$

- $$\begin{aligned} \psi^f &= (E_x, E_y, iE_z, H_x, H_y, iH_z)^\top \\ \psi^b &= (E_x, E_y, -iE_z, -H_x, -H_y, iH_z)^\top. \end{aligned}$$

(Real components).

$$[\psi_\nu^f; \psi_\mu^f] = [\psi_\nu^b; \psi_\mu^b] = 0, \quad [\psi_\nu^f; \psi_\mu^b] = -\delta_{\nu\mu} 4P_\nu, \quad [\psi_\nu^b; \psi_\mu^f] = \delta_{\nu\mu} 4P_\nu.$$

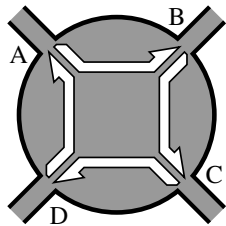
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Nonreciprocal devices



Isolator:
unidirectional transmission,
 $S_{BA} = 1, S_{AB} = 0.$



Circulator:
transmission cycle,
 $S_{BA} = 1, S_{CB} = 1, S_{DC} = 1, S_{AD} = 1,$
 $S_{..} = 0$ otherwise.

Required: nonreciprocal media with $\hat{\epsilon} \neq \hat{\epsilon}^\top$,
↔ magnetooptic media, Faraday effect.

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Scattering matrices, reciprocity

$$\hookrightarrow 0 = \sum_\nu 4P_\nu (B_{1,\nu} F_{2,\nu} - F_{1,\nu} B_{2,\nu}),$$

uniform normalization $P_\nu = P_0$,

$$\hookrightarrow 0 = \sum_\nu (B_{1,\nu} F_{2,\nu} - F_{1,\nu} B_{2,\nu}),$$

$$\hookrightarrow 0 = \mathbf{B}_1 \cdot \mathbf{F}_2 - \mathbf{F}_1 \cdot \mathbf{B}_2,$$

$$\mathbf{F}_j = \mathbf{S} \mathbf{B}_j,$$

$$\hookrightarrow 0 = \mathbf{B}_1 \cdot \mathbf{S} \mathbf{B}_2 - (\mathbf{S} \mathbf{B}_1) \cdot \mathbf{B}_2,$$

$$\hookrightarrow 0 = \mathbf{B}_1 \cdot \mathbf{S} \mathbf{B}_2 - \mathbf{B}_1 \cdot \mathbf{S}^\top \mathbf{B}_2,$$

$$\hookrightarrow 0 = \mathbf{B}_1 \cdot (\mathbf{S} - \mathbf{S}^\top) \mathbf{B}_2 \text{ for all } \mathbf{B}_1, \mathbf{B}_2.$$

$$\mathbf{S} = \mathbf{S}^\top, \quad S_{\nu\mu} = S_{\mu\nu} \text{ for all } \nu, \mu.$$

The scattering matrix of a *reciprocal circuit* is *symmetric*.

Reciprocal circuit: made of reciprocal media, with $\hat{\epsilon} = \hat{\epsilon}^\top, \hat{\mu} = \hat{\mu}^\top$.

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Nonreciprocal devices

What about, for example,

- a long, “adiabatic” Y-junction ?
- a junction between a single mode core and a wider multimode waveguide ?



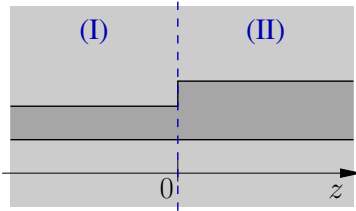
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Waveguide discontinuities



Half-infinite waveguides (I), (II), discontinuity at $z = 0$.

- Expand into local normal modes $\{\psi_{s,m}^d, \beta_{s,m}\}$, $m \in \mathcal{N}_s$, $s = \text{I, II}$:
Transverse boundary conditions \leftrightarrow discrete sets.

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_s(x, y, z) = \sum_{m \in \mathcal{N}_s} \left\{ f_{s,m} \psi_{s,m}^f(x, y) e^{-i\beta_{s,m}z} + b_{s,m} \psi_{s,m}^b(x, y) e^{+i\beta_{s,m}z} \right\},$$

$z < 0$: $s = \text{I}$, $f_{\text{I},m}$ given influx, $b_{\text{I},m}$ unknown,
 $z > 0$: $s = \text{II}$, $f_{\text{II},m}$ unknown, $b_{\text{II},m}$ given influx.

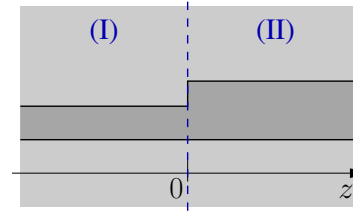
\hookrightarrow $(\mathbf{E}, \mathbf{H})_{\text{I,II}}$ are solutions for $z < 0$ and $z > 0$.

- Continuity of the tangential components of \mathbf{E}, \mathbf{H} at the interface \leftrightarrow formally equate expressions for $(\mathbf{E}, \mathbf{H})_{\text{I,II}}$ at $z = 0$.

(Only equality of E_x, E_y, H_x, H_y will be relevant.)

- Project on $\psi_{s,l}^d$ to extract coefficients ...

Waveguide discontinuities, scattering matrix

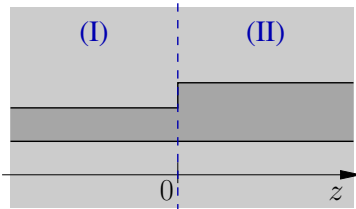


(Global coordinate $z \neq$ former local coordinate on port I.)
(One variant of a projection procedure.)

- $(\psi_{\text{I},l}^b; \cdot = \cdot)$, $l \in \mathcal{N}_{\text{I}}$:
$$\sum_{m \in \mathcal{N}_{\text{I}}} [f_{\text{I},m}(\psi_{\text{I},l}^b; \psi_{\text{I},m}^f) + b_{\text{I},m}(\psi_{\text{I},l}^b; \psi_{\text{I},m}^b)] = \sum_{m \in \mathcal{N}_{\text{II}}} [f_{\text{II},m}(\psi_{\text{I},l}^b; \psi_{\text{II},m}^f) + b_{\text{II},m}(\psi_{\text{I},l}^b; \psi_{\text{II},m}^b)],$$
- $(\psi_{\text{II},l}^f; \cdot = \cdot)$, $l \in \mathcal{N}_{\text{II}}$:
$$\sum_{m \in \mathcal{N}_{\text{I}}} [f_{\text{I},m}(\psi_{\text{II},l}^f; \psi_{\text{I},m}^f) + b_{\text{I},m}(\psi_{\text{II},l}^f; \psi_{\text{I},m}^b)] = \sum_{m \in \mathcal{N}_{\text{II}}} [f_{\text{II},m}(\psi_{\text{II},l}^f; \psi_{\text{II},m}^f) + b_{\text{II},m}(\psi_{\text{II},l}^f; \psi_{\text{II},m}^b)],$$

$$\hookrightarrow \dots \rightsquigarrow \begin{pmatrix} \mathbf{b}_{\text{I}} \\ \mathbf{b}_{\text{II}} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \mathbf{f}_{\text{I}} \\ \mathbf{b}_{\text{II}} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{\text{I,I}} & \mathbf{S}_{\text{I,II}} \\ \mathbf{S}_{\text{II,I}} & \mathbf{S}_{\text{II,II}} \end{pmatrix} \begin{pmatrix} \mathbf{f}_{\text{I}} \\ \mathbf{b}_{\text{II}} \end{pmatrix}.$$

Waveguide discontinuities, overlap model



Most simplified variant:
Unidirectional **overlap model**.

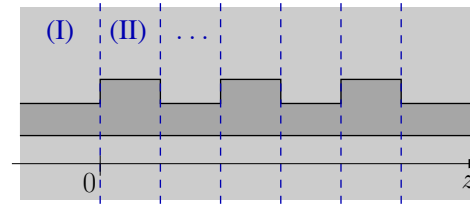
- (I): Incoming guided mode ψ_{I} , reflections & radiation neglected.
(II): Outgoing guided modes $\psi_{\text{II},m}$, radiation neglected.

$$f_{\text{I}} \psi_{\text{I}} \approx \sum_m f_{\text{II},m} \psi_{\text{II},m} \text{ at } z = 0.$$

$$\hookrightarrow f_{\text{II},m} = \frac{(\psi_{\text{II},m}; \psi_{\text{I}})}{(\psi_{\text{II},m}; \psi_{\text{II},m})} f_{\text{I}}, \quad \text{or} \quad f_{\text{II},m} = \frac{1}{P_{\text{II},m}} (\psi_{\text{II},m}; \psi_{\text{I}}) f_{\text{I}}.$$

(Transmission is given directly by the "overlaps" \leftrightarrow Relevance of the mode products $(\cdot; \cdot)$.
(Cf. explicit expressions for overlaps of 2-D modes, involving only principal mode profile components.)

A sequence of waveguide discontinuities



- Divide into segments.
- Establish local normal mode expansions.
- Project on local modes.

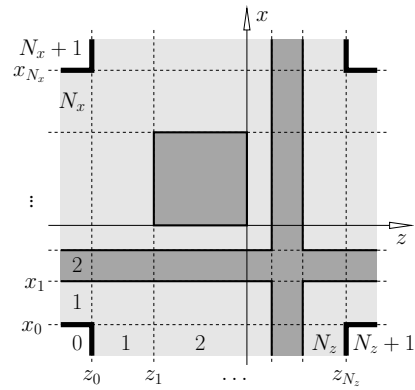
\hookrightarrow Linear system of equations for all local mode amplitudes.

$$\hookrightarrow \text{Solve } (\dots) \rightsquigarrow \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(x, y, z).$$

Bidirectional eigenmode propagation (BEP),
Eigenmode expansion method (EME),
...

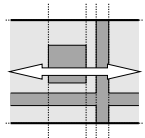
(Radiated outgoing fields: Open boundary conditions required (PMLs) \leftrightarrow Complex eigenmodes.)
(2-D: ok. 3-D: ?)

Rectangular 2-D circuits

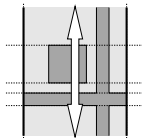


Quadridirectional Eigenmode Propagation (QUEP)

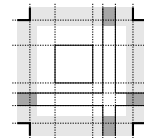
- Divide into slices & layers.
- Establish local modes:
Propagation along $\pm z$,
& Propagation along $\pm x$,
boundary conditions $\phi = 0$.
- Project at horizontal
& vertical interfaces.



horizontal BEP,



vertical BEP,



continuity at x_0, x_N, z_0, z_{N_z} .

Upcoming

Next lectures:

- Bent optical waveguides; whispering gallery resonances; circular microresonators.
- Coupled mode theory, perturbation theory.
- A touch of photonic crystals; a touch of plasmonics.

