# *Optical Waveguide Theory (G)*

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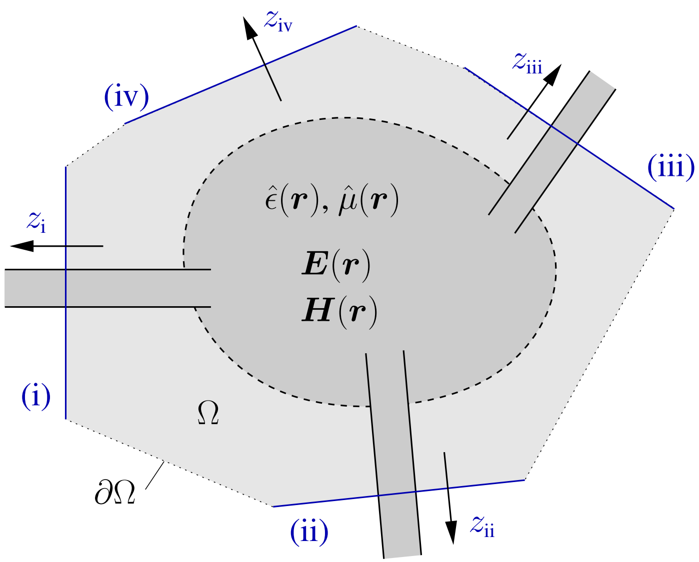
## *Course overview*

### Optical waveguide theory





## *PICs, OICs, scattering matrices*






## *Scattering matrices, prerequisites*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)

- Passive, linear circuit.
- (Computational) domain of interest $\Omega$, its boundary $\partial\Omega$.
- Connecting channels: lossless waveguides (or "half-spaces").
- Physical ports $p = \mathrm{i}, \mathrm{ii}, \dots$: waveguide cross-section planes, local coordinates $x_p, y_p, z_p$; local axis $z_p$ oriented outwards of $\Omega$.
- Establish sets $\mathcal{N}_p$ of *propagating* directional normal modes $\{\psi^d_{p,m} := (\boldsymbol{E}^d_{p,m}, \boldsymbol{H}^d_{p,m}),\ \beta_{p,m};\ d = \mathrm{f,b}\}$ on each port $p$.

(Restriction to propagating fields: a condition on port positioning / a model assumption.)

- Ports & modes are such that all mode fields vanish on all "other" port planes, and on $\partial\Omega$ outside the ports.

Field on port plane $p$ and "outside":

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x_p, y_p, z_p) = \sum_{m\in\mathcal{N}_p} F_{p,m}\psi^{\mathrm{f}}_{p,m}(x_p, y_p)\ \mathrm{e}^{-\mathrm{i}\beta_{p,m}z_p} + B_{p,m}\psi^{\mathrm{b}}_{p,m}(x_p, y_p)\ \mathrm{e}^{\mathrm{i}\beta_{p,m}z_p}.$$

## *Scattering matrices*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)

- Merge all mode indices $\{m\}$ and port IDs $\{p\}$ into one set of mode identifiers $\{\nu\}$, $\quad \mathcal{N} = \cup_p \mathcal{N}_p$.
- Assert that $\psi^{\cdot}_{p,\cdot}(\boldsymbol{r}) = 0$ for all $\boldsymbol{r} \in \partial\Omega$, $\boldsymbol{r} \notin$ port $p$.
- Field on $\partial\Omega$: $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_{\nu \in \mathcal{N}} \{F_\nu \psi^{\mathrm{f}}_\nu + B_\nu \psi^{\mathrm{b}}_\nu\}.$

(Position arguments omitted.)

- $B_\nu$ : $\sim$ incident modes, traveling towards the interior of $\Omega$.
  $F_\nu$ : $\sim$ outgoing modes, traveling towards the exterior of $\Omega$.
  Combine into amplitude vectors $\boldsymbol{B}, \boldsymbol{F}$.

---

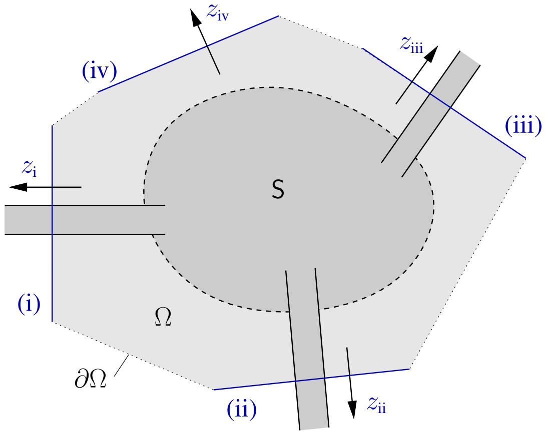
Linear circuit $\longleftrightarrow$ linear dependence of $\boldsymbol{F}$ on $\boldsymbol{B}$,
Scattering matrix $\mathsf{S}$ of the circuit: $\quad \boldsymbol{F} = \mathsf{S}\,\boldsymbol{B}, \qquad \mathsf{S} = (S_{\nu\mu})$.

- $S_{\nu\nu}$ : $\sim (\nu, \mathrm{b}) \to (\nu, \mathrm{f})$, reflection coefficient for mode $\nu$.
- $S_{\nu\mu}$ : $\sim (\mu, \mathrm{b}) \to (\nu, \mathrm{f})$, transmission coefficient for modes $\mu, \nu$.



## *PICs, OICs, scattering matrices, scenarios*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)

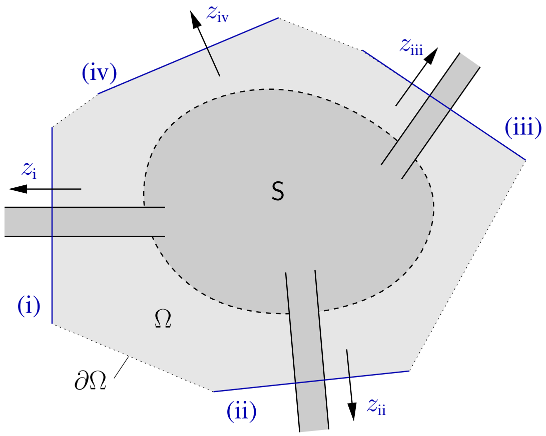



- Scenario: Full matrix $\mathsf{S}$, including guided and radiation modes, large $\dim \mathsf{S}$ $\leftrightarrow$ theoretical results.



## *PICs, OICs, scattering matrices, scenarios*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)



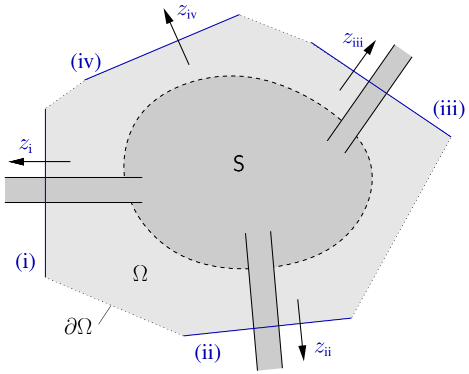

- Scenario: Restrict to a specific set of (guided) modes, or:
  Only a small set of guided modes are relevant:
  small $\dim \mathsf{S} = N \times N$ $\leftrightarrow$ an $N$-port circuit, a 2-$N$-pole.

($N$: the total number of relevant modes, not the number of ports.)



## *Scattering matrices, port plane positions*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)



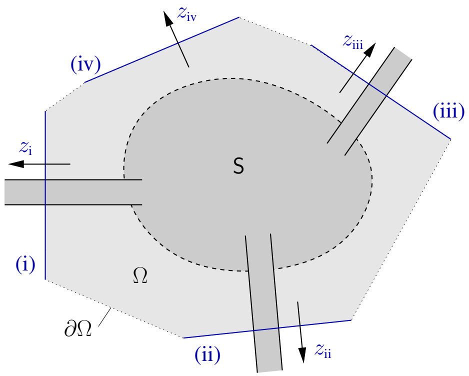

- Shift port plane of mode $\nu$ by $\Delta z_\nu$: $\quad F_\nu \;\to\; F'_\nu = F_\nu\,\mathrm{e}^{-\mathrm{i}\beta_\nu \Delta z_\nu}$,
  Shift port plane of mode $\mu$ by $\Delta z_\mu$: $\quad B_\mu \;\to\; B'_\mu = B_\mu\,\mathrm{e}^{\mathrm{i}\beta_\mu \Delta z_\mu}$,

$$\hookrightarrow \quad F'_\nu = S'_{\nu\mu} B'_\mu, \quad S'_{\nu\mu} = S_{\nu\mu}\,\mathrm{e}^{-\mathrm{i}(\beta_\nu \Delta z_\nu + \beta_\mu \Delta z_\mu)}.$$

(Moving port planes $\leftrightarrow$ Phase change in reflection / transmission coefficients.)
(Moving port planes $\leftrightarrow$ No effect on reflectances / transmittances.)

## *Scattering matrices, port mode orthogonality*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)




- Orthogonality relations on port plane $p$:

$$(\boldsymbol{E}_a, \boldsymbol{H}_a; \boldsymbol{E}_b, \boldsymbol{H}_b) = \frac{1}{4} \iint_p \left(E^*_{ax} H_{by} - E^*_{ay} H_{bx} + H^*_{ay} E_{bx} - H^*_{ax} E_{by}\right) \mathrm{d}x_p\, \mathrm{d}y_p$$

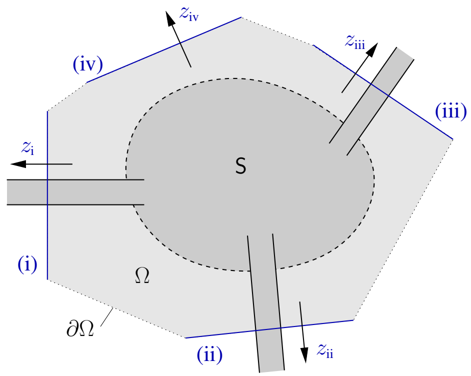
$$(\psi^d_{p,l}; \psi^r_{p,m}) = \pm \delta_{dr} \delta_{lm} P_{p,m}.$$

(Things restricted to propagating modes.)



## *Scattering matrices, port mode orthogonality*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)




- Extend to the full boundary $\partial\Omega$:

$$(\boldsymbol{E}_a, \boldsymbol{H}_a; \boldsymbol{E}_b, \boldsymbol{H}_b) := \frac{1}{4} \int_{\partial\Omega} (\boldsymbol{E}^*_a \times \boldsymbol{H}_b + \boldsymbol{E}_b \times \boldsymbol{H}^*_a) \cdot \mathrm{d}\boldsymbol{a}$$

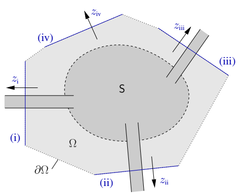
$$(\psi^d_{p,l}; \psi^r_{q,m}) = \pm \delta_{dr} \delta_{pq} \delta_{lm} P_{p,m} \quad \text{or} \quad (\psi^d_{\nu}; \psi^r_{\mu}) = \pm \delta_{dr} \delta_{\nu\mu} P_{\nu}.$$

(Modes belonging to different ports are mutually orthogonal.)



## *Scattering matrices, power balance*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)




- Net power outflow across the border of the circuit:

$$P = \int_{\partial\Omega} \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = (\boldsymbol{E}, \boldsymbol{H}; \boldsymbol{E}, \boldsymbol{H}) = \sum_p \sum_{m \in \mathcal{N}_p} \left(|F_{p,m}|^2 - |B_{p,m}|^2\right) P_{p,m}$$
$$= \sum_{\nu \in \mathcal{N}} \left(|F_\nu|^2 - |B_\nu|^2\right) P_\nu\,,$$

$|B_\mu|^2 P_\mu$: incident power carried by mode $\mu$,
$|F_\nu|^2 P_\nu$: outgoing power carried by mode $\nu$, $\qquad F_\nu = S_{\nu\mu} B_\mu$.

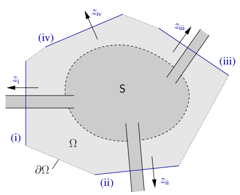
$$|S_{\nu\mu}|^2 \frac{P_\nu}{P_\mu} = \frac{|F_\nu|^2 P_\nu}{|B_\mu|^2 P_\mu}, \quad \begin{array}{ll} \mu \neq \nu: & \text{power transmittance } \mu \to \nu, \\ \mu = \nu: & \text{power reflectance for mode } \nu. \end{array}$$

(Uniform normalized modes, $P_\nu = P_\mu$: transmittances are directly given by elements of the scattering matrix).



## *Scattering matrices, power balance*

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)




- Net power outflow across the border of the circuit:

$$P = \int_{\partial\Omega} \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = (\boldsymbol{E}, \boldsymbol{H}; \boldsymbol{E}, \boldsymbol{H}) = P_0 \left(\boldsymbol{B}^* \cdot (\mathsf{S}^\dagger \mathsf{S} - 1) \boldsymbol{B}\right),$$

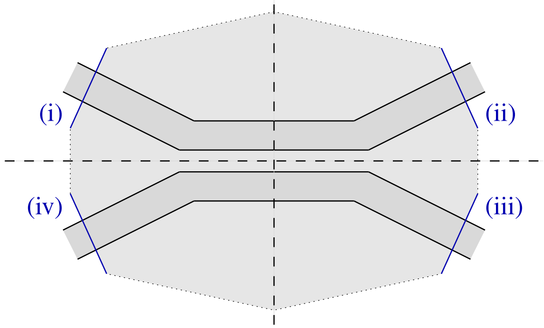
uniform normalization, $P_\nu = P_0$ for all $\nu$.

- Lossless circuit $\longleftrightarrow$ $\int_{\partial\Omega} \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = 0$ $\longleftrightarrow$ $\mathsf{S}^\dagger \mathsf{S} = 1$,
  the scattering matrix of a lossless circuit is unitary.

- Lossy circuit $\longleftrightarrow$ $\int_{\partial\Omega} \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} \leq 0$ $\rightsquigarrow$ $\boldsymbol{B}^* \cdot \mathsf{S}^\dagger \mathsf{S} \boldsymbol{B} \leq \boldsymbol{B}^* \boldsymbol{B}$,
  $\sum_\nu |S_{\nu\mu}|^2 \leq 1$ for all $\mu$.

(The sum of transmittances mode $\mu$ to all other modes $\nu$ is less than one.)
(Interior lossy media, or radiative losses: outgoing propagating modes not taken into account.)

## *Scattering matrices, symmetry*




Circuit with specific spatial symmetry

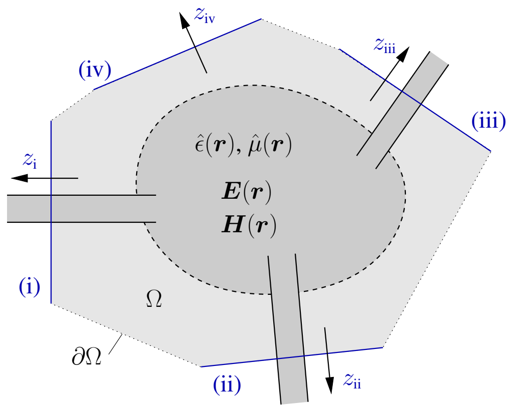
& symmetrical setting of the port planes

respective symmetry in related coefficients of S,
symmetric power transmission properties.



## *Scattering matrices, reciprocity*

$\sim \exp(\mathrm{i}\omega t)$ (FD)




Circuit properties
for reversed
wave propagation ?

$S_{\nu\mu} \leftrightarrow S_{\mu\nu}$ ?

- $\boldsymbol{E}_1, \boldsymbol{H}_1$ and $\boldsymbol{E}_2, \boldsymbol{H}_2$ solve $\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0\hat{\mu}\boldsymbol{H}$, $\boldsymbol{\nabla} \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\hat{\epsilon}\boldsymbol{E}$.

  $\boldsymbol{\nabla} \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{H}_1 \times \boldsymbol{E}_2) = 0$, if $\hat{\epsilon}$ and $\hat{\mu}$ are symmetric.

(i.e. if $\hat{\epsilon}^{\mathsf{T}} = \epsilon$, $\hat{\mu}^{\mathsf{T}} = \mu$.)
(Note: order of factors, no complex conjugates.)



## *Scattering matrices, reciprocity*

- $\boldsymbol{E}_1, \boldsymbol{H}_1$ and $\boldsymbol{E}_2, \boldsymbol{H}_2$ solve $\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0\hat{\mu}\boldsymbol{H}$, $\boldsymbol{\nabla} \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\hat{\epsilon}\boldsymbol{E}$

  $\boldsymbol{\nabla} \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{H}_1 \times \boldsymbol{E}_2) = 0$, if $\hat{\epsilon}$ and $\hat{\mu}$ are symmetric,

$$0 = \int_\Omega \boldsymbol{\nabla} \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{H}_1 \times \boldsymbol{E}_2)\, \mathrm{d}^3 r = \int_{\partial\Omega} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{H}_1 \times \boldsymbol{E}_2) \cdot \mathrm{d}\boldsymbol{a}.$$

- Fields on $\partial\Omega$: $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_j = \sum_{\nu\in\mathcal{N}} \{F_{j,\nu}\psi_\nu^{\mathrm{f}} + B_{j,\nu}\psi_\nu^{\mathrm{b}}\}$, $\quad j = 1, 2$,

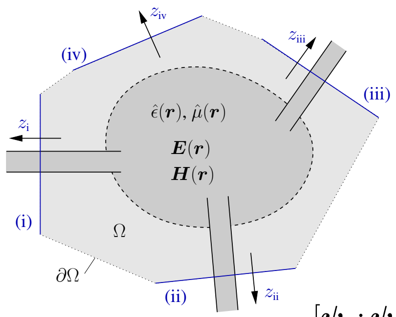
  $$[\boldsymbol{\psi}_a; \boldsymbol{\psi}_b] := \int_{\partial\Omega} (\boldsymbol{E}_a \times \boldsymbol{H}_b + \boldsymbol{H}_a \times \boldsymbol{E}_b) \cdot \mathrm{d}\boldsymbol{a},$$

  $$0 = \sum_\nu \sum_\mu \big( F_{1,\nu}F_{2,\mu}[\boldsymbol{\psi}_\nu^{\mathrm{f}}; \boldsymbol{\psi}_\mu^{\mathrm{f}}] + F_{1,\nu}B_{2,\mu}[\boldsymbol{\psi}_\nu^{\mathrm{f}}; \boldsymbol{\psi}_\mu^{\mathrm{b}}] + B_{1,\nu}F_{2,\mu}[\boldsymbol{\psi}_\nu^{\mathrm{b}}; \boldsymbol{\psi}_\mu^{\mathrm{f}}] + B_{1,\nu}B_{2,\mu}[\boldsymbol{\psi}_\nu^{\mathrm{b}}; \boldsymbol{\psi}_\mu^{\mathrm{b}}] \big).$$



## *Scattering matrices, reciprocity*

$\sim \exp(\mathrm{i}\omega t)$ (FD)




$$[\boldsymbol{\psi}_a; \boldsymbol{\psi}_b] := \int_{\partial\Omega} (\boldsymbol{E}_a \times \boldsymbol{H}_b + \boldsymbol{H}_a \times \boldsymbol{E}_b) \cdot \mathrm{d}\boldsymbol{a}.$$

- $[\boldsymbol{\psi}_\nu^{\cdot}; \boldsymbol{\psi}_\mu^{\cdot}] = 0$, if $\nu$ and $\mu$ relate to different ports.
- If $\nu$ and $\mu$ relate to the same port plane $p$:

  $$[\boldsymbol{\psi}_\nu^r; \boldsymbol{\psi}_\mu^d] = \iint_p \big(E_{\nu x}^r H_{\mu y}^d - E_{\nu y}^r H_{\mu x}^d - H_{\nu y}^r E_{\mu x}^d + H_{\nu x}^r E_{\mu y}^d\big)\, \mathrm{d}x_p\, \mathrm{d}y_p.$$

## *Scattering matrices, reciprocity*

- If $\nu$ and $\mu$ relate to the same port plane $p$:
  $$[\psi^r_\nu;\psi^d_\mu] = \iint_p \big(E^r_{\nu x}H^d_{\mu y} - E^r_{\nu y}H^d_{\mu x} - H^r_{\nu y}E^d_{\mu x} + H^r_{\nu x}E^d_{\mu y}\big)\,\mathrm{d}x_p\,\mathrm{d}y_p.$$

- Compare with the modal orthogonality relations on port plane $p$, for propagating modes with real transverse components:
  $$(\psi^r_\nu;\psi^d_\mu) = \frac{1}{4}\iint_p \big(E^r_{\nu x}H^d_{\mu y} - E^r_{\nu y}H^d_{\mu x} + H^r_{\nu y}E^d_{\mu x} - H^r_{\nu x}E^d_{\mu y}\big)\,\mathrm{d}x_p\,\mathrm{d}y_p,$$
  $$(\psi^\mathrm{f}_\nu;\psi^\mathrm{f}_\mu) = \delta_{\nu\mu}P_\nu,\quad (\psi^\mathrm{b}_\nu;\psi^\mathrm{b}_\mu) = -\delta_{\nu\mu}P_\nu,\quad (\psi^\mathrm{f}_\nu;\psi^\mathrm{b}_\mu) = (\psi^\mathrm{b}_\nu;\psi^\mathrm{f}_\mu) = 0.$$

- $$\psi^\mathrm{f} = (E_x, E_y, \;\mathrm{i}E_z, \;\; H_x, \;\; H_y, \mathrm{i}H_z)^\mathsf{T}$$
  $$\leftrightarrow \quad \psi^\mathrm{b} = (E_x, E_y, -\mathrm{i}E_z, \; -H_x, -H_y, \mathrm{i}H_z)^\mathsf{T}. \qquad \text{(Real components).}$$

$$[\psi^\mathrm{f}_\nu;\psi^\mathrm{f}_\mu] = [\psi^\mathrm{b}_\nu;\psi^\mathrm{b}_\mu] = 0,\quad [\psi^\mathrm{f}_\nu;\psi^\mathrm{b}_\mu] = -\delta_{\nu\mu}4P_\nu,\quad [\psi^\mathrm{b}_\nu;\psi^\mathrm{f}_\mu] = \delta_{\nu\mu}4P_\nu.$$



## *Scattering matrices, reciprocity*

$$\hookrightarrow \quad 0 = \sum_\nu 4P_\nu\big(B_{1,\nu}F_{2,\nu} - F_{1,\nu}B_{2,\nu}\big),$$

uniform normalization $P_\nu = P_0$,

$$\hookrightarrow \quad 0 = \sum_\nu \big(B_{1,\nu}F_{2,\nu} - F_{1,\nu}B_{2,\nu}\big),$$

$$\hookrightarrow \quad 0 = \boldsymbol{B}_1\cdot\boldsymbol{F}_2 - \boldsymbol{F}_1\cdot\boldsymbol{B}_2, \qquad \boldsymbol{F}_j = \mathsf{S}\,\boldsymbol{B}_j,$$

$$\hookrightarrow \quad 0 = \boldsymbol{B}_1\cdot\mathsf{S}\,\boldsymbol{B}_2 - (\mathsf{S}\,\boldsymbol{B}_1)\cdot\boldsymbol{B}_2,$$

$$\hookrightarrow \quad 0 = \boldsymbol{B}_1\cdot\mathsf{S}\,\boldsymbol{B}_2 - \boldsymbol{B}_1\cdot\mathsf{S}^\mathsf{T}\boldsymbol{B}_2,$$

$$\hookrightarrow \quad 0 = \boldsymbol{B}_1\cdot\big(\mathsf{S} - \mathsf{S}^\mathsf{T}\big)\boldsymbol{B}_2 \quad \text{for all } \boldsymbol{B}_1, \boldsymbol{B}_2.$$

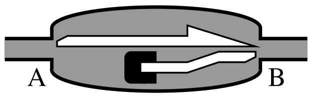
$$\mathsf{S} = \mathsf{S}^\mathsf{T}, \quad S_{\nu\mu} = S_{\mu\nu} \quad \text{for all } \nu, \mu.$$

*The scattering matrix of a reciprocal circuit is symmetric.*

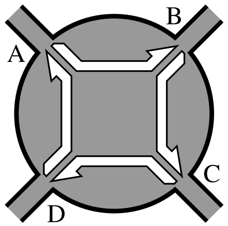
Reciprocal circuit: made of reciprocal media, with $\hat\epsilon = \hat\epsilon^\mathsf{T}$, $\hat\mu = \hat\mu^\mathsf{T}$.



## *Nonreciprocal devices*




Isolator:
unidirectional transmission,
$S_\mathrm{BA} = 1,\quad S_\mathrm{AB} = 0.$




Circulator:
transmission cycle,
$S_\mathrm{BA} = 1,\quad S_\mathrm{CB} = 1,\quad S_\mathrm{DC} = 1,\quad S_\mathrm{AD} = 1,$
$S_{\cdot\cdot} = 0$ otherwise.

Required: nonreciprocal media with $\hat\epsilon \neq \hat\epsilon^\mathsf{T}$,
$\leftrightarrow$ magnetooptic media, Faraday effect.

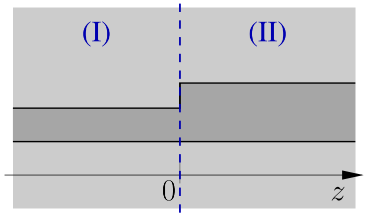


## *Nonreciprocal devices*

What about, for example,

- a long, “adiabatic” Y-junction ?
- a junction between a single mode core and a wider multimode waveguide ?

## *Waveguide discontinuities*




Half-infinite waveguides (I), (II), discontinuity at $z = 0$.

- Expand into local normal modes $\{\psi^d_{s,m}, \beta_{s,m}\}$, $m \in \mathcal{N}_s$, $s = \mathrm{I}, \mathrm{II}$:

Transverse boundary conditions $\rightsquigarrow$ discrete sets.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_s (x,y,z) = \sum_{m\in\mathcal{N}_s} \left\{ f_{s,m} \psi^{\mathrm{f}}_{s,m}(x,y)\, \mathrm{e}^{-\mathrm{i}\beta_{s,m}z} + b_{s,m} \psi^{\mathrm{b}}_{s,m}(x,y)\, \mathrm{e}^{+\mathrm{i}\beta_{s,m}z} \right\},$$

$z < 0$: $s = \mathrm{I}$, $f_{\mathrm{I},m}$ given influx, $b_{\mathrm{I},m}$ unknown,
$z > 0$: $s = \mathrm{II}$, $f_{\mathrm{II},m}$ unknown, $b_{\mathrm{II},m}$ given influx.

$\hookrightarrow$ $(\boldsymbol{E}, \boldsymbol{H})_{\mathrm{I,II}}$ are solutions for $z < 0$ and $z > 0$.

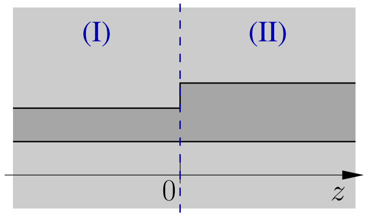
- Continuity of the tangential components of $\boldsymbol{E}, \boldsymbol{H}$ at the interface
  $\rightsquigarrow$ formally equate expressions for $(\boldsymbol{E}, \boldsymbol{H})_{\mathrm{I,II}}$ at $z = 0$.

(Only equality of $E_x$, $E_y$, $H_x$, $H_y$ will be relevant.)

- Project on $\psi^d_{s,l}$ to extract coefficients ...



## *Waveguide discontinuities, scattering matrix*




(Global coordinate $z \neq$ former local coordinate on port I.)
(One variant of a projection procedure.)

- $(\psi^{\mathrm{b}}_{\mathrm{I},l};\ \cdot = \cdot\ )$, $l \in \mathcal{N}_{\mathrm{I}}$:

$$\sum_{m\in\mathcal{N}_{\mathrm{I}}} \left[ f_{\mathrm{I},m} (\psi^{\mathrm{b}}_{\mathrm{I},l}; \psi^{\mathrm{f}}_{\mathrm{I},m}) + b_{\mathrm{I},m} (\psi^{\mathrm{b}}_{\mathrm{I},l}; \psi^{\mathrm{b}}_{\mathrm{I},m}) \right] = \sum_{m\in\mathcal{N}_{\mathrm{II}}} \left[ f_{\mathrm{II},m} (\psi^{\mathrm{b}}_{\mathrm{I},l}; \psi^{\mathrm{f}}_{\mathrm{II},m}) + b_{\mathrm{II},m} (\psi^{\mathrm{b}}_{\mathrm{I},l}; \psi^{\mathrm{b}}_{\mathrm{II},m}) \right],$$

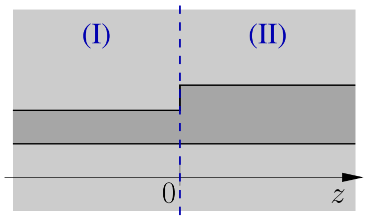
- $(\psi^{\mathrm{f}}_{\mathrm{II},l};\ \cdot = \cdot\ )$, $l \in \mathcal{N}_{\mathrm{II}}$:

$$\sum_{m\in\mathcal{N}_{\mathrm{I}}} \left[ f_{\mathrm{I},m} (\psi^{\mathrm{f}}_{\mathrm{II},l}; \psi^{\mathrm{f}}_{\mathrm{I},m}) + b_{\mathrm{I},m} (\psi^{\mathrm{f}}_{\mathrm{II},l}; \psi^{\mathrm{b}}_{\mathrm{I},m}) \right] = \sum_{m\in\mathcal{N}_{\mathrm{II}}} \left[ f_{\mathrm{II},m} (\psi^{\mathrm{f}}_{\mathrm{II},l}; \psi^{\mathrm{f}}_{\mathrm{II},m}) + b_{\mathrm{II},m} (\psi^{\mathrm{f}}_{\mathrm{II},l}; \psi^{\mathrm{b}}_{\mathrm{II},m}) \right],$$

$\hookrightarrow$ ... $\rightsquigarrow$ $\begin{pmatrix} \boldsymbol{b}_{\mathrm{I}} \\ \boldsymbol{f}_{\mathrm{II}} \end{pmatrix} = \mathsf{S} \begin{pmatrix} \boldsymbol{f}_{\mathrm{I}} \\ \boldsymbol{b}_{\mathrm{II}} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\mathrm{I,I}} & \mathsf{S}_{\mathrm{I,II}} \\ \mathsf{S}_{\mathrm{II,I}} & \mathsf{S}_{\mathrm{II,II}} \end{pmatrix} \begin{pmatrix} \boldsymbol{f}_{\mathrm{I}} \\ \boldsymbol{b}_{\mathrm{II}} \end{pmatrix}.$



## *Waveguide discontinuities, overlap model*




Most simplified variant:
Unidirectional overlap model.

- (I): Incoming guided mode $\psi_{\mathrm{I}}$, reflections & radiation neglected.
  (II): Outgoing guided modes $\psi_{\mathrm{II},m}$, radiation neglected.

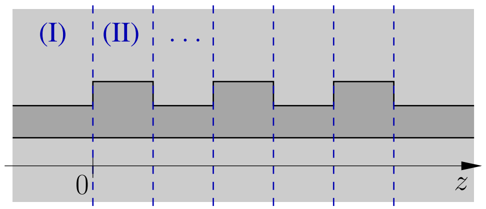
- $f_{\mathrm{I}}\, \psi_{\mathrm{I}} \approx \sum_m f_{\mathrm{II},m}\, \psi_{\mathrm{II},m}$ at $z = 0$.

$\hookrightarrow$ $f_{\mathrm{II},m} = \dfrac{(\psi_{\mathrm{II},m}; \psi_{\mathrm{I}})}{(\psi_{\mathrm{II},m}; \psi_{\mathrm{II},m})} f_{\mathrm{I}}$, or $f_{\mathrm{II},m} = \dfrac{1}{P_{\mathrm{II},m}} (\psi_{\mathrm{II},m}; \psi_{\mathrm{I}})\, f_{\mathrm{I}}$.

(Transmission is given directly by the "overlaps" $\rightsquigarrow$ Relevance of the mode products $(\,\cdot\,;\,\cdot\,)$.)
(Cf. explicit expressions for overlaps of 2-D modes, involving only principal mode profile components.)



## *A sequence of waveguide discontinuities*




- Divide into segments.
- Establish local normal mode expansions.
- Project on local modes.

$\hookrightarrow$ Linear system of equations for all local mode amplitudes.

$\hookrightarrow$ Solve ( ... ) $\rightsquigarrow$ $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (x,y,z)$.

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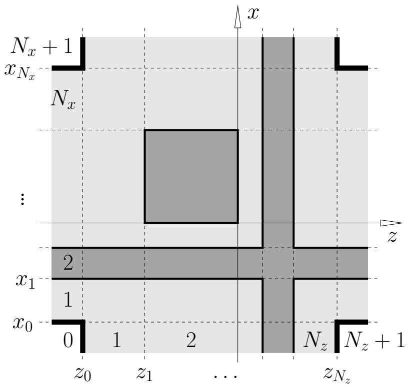
*Bidirectional eigenmode propagation* (BEP),
*Eigenmode expansion method* (EME),
... .

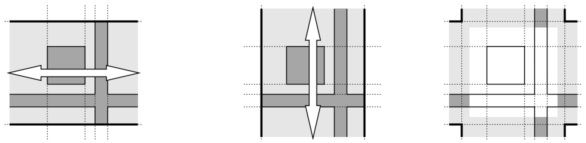
(Radiated outgoing fields: Open boundary conditions required (PMLs) $\rightsquigarrow$ Complex eigenmodes.)
(2-D: ok. 3-D: ?)

## *Rectangular 2-D circuits*




*Quadridirectional Eigenmode Propagation* (QUEP)

- Divide into slices & layers.
- Establish local modes:
  Propagation along $\pm z$,
  & Propagation along $\pm x$,
  boundary conditions $\phi = 0$.
- Project at horizontal & vertical interfaces.



↪ horizontal BEP, vertical BEP, continuity at $x_0, x_N, z_0, z_N$.



## *Upcoming*

Next lectures:

- Bent optical waveguides; whispering gallery resonances; circular microresonators.
- Coupled mode theory, perturbation theory.
- A touch of photonic crystals; a touch of plasmonics.