Course overview

Optical waveguide theory

A Photonic / integrated optics; theory, motto; phenomena, introductory examples.
B Maxwell equations, different formulations, interfaces, energy and power flow.
C Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
D Normal modes of dielectric optical waveguides, mode interference.
E Examples for dielectric optical waveguides.
F Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
H Bent optical waveguides; whispering gallery resonances; circular microresonators.
J A touch of photonic crystals; a touch of plasmonics.
• Oblique semi-guided waves: 2-D integrated optics.
• Summary, concluding remarks.

A touch of photonic crystals

“Photonic crystals”: ?

Keywords:
- A branch of photonics.
- Optics involving structures with (1-D, 2-D, 3-D) spatial periodicity.
- 1-D periodicity: Multilayer stacks / coatings, gratings, corrugated waveguides.
- 2-D periodicity: Corrugated dielectric slabs, membranes, gratings.
- 3-D periodicity: Bulk photonic crystals.
- "Molding the flow of light" tunability, degrees of freedom in design.
- Defect cavities & defect waveguides in photonic crystals.
- Phenomena & fundamental research.
- Photonic crystal fibers.

Context of this lecture:
- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Periodicity: Restrict computations to unit cells.

Structures with spatial periodicity

Infinite system with periodic permittivity:

$$\epsilon(r + g) = \epsilon(r) \quad \text{for all lattice vectors } g.$$  

Consider Floquet-Bloch waves

$$\begin{bmatrix} E \\ H \end{bmatrix}(r) = U_k(r) \ e^{-i k \cdot r},$$

$$k: \text{ wavevector of the FB wave},$$

$$U_k: \text{ a periodic function, } U_k(r + g) = U_k(r).$$  

(A plane wave, modulated by a periodic function.)

{FB waves}: A complete basis for the periodic system.

(Bloch theorem: any solution can be written as a superposition of FB waves.)
(Background: Hilbert space theory, self-adjoint operators, familiar from Quantum theory.)
(Hermitian Hamiltonian and translation operators commute; Bloch waves are a simultaneous eigenbasis of these operators.)
(REquired: Hermitian "Hamiltonian" $\hat{\epsilon}$.)

($U_k = ?$, but $U_k$ satisfies different equations than $E, H \ldots$)
Structures with spatial periodicity

\[ g : \text{a lattice vector, such that } \epsilon(r + g) = \epsilon(r) \sim \exp(i \omega t) \ (\text{FD}) \]

\[
\begin{pmatrix}
E \\
H
\end{pmatrix}(r + g) = \begin{pmatrix}
E \\
H
\end{pmatrix}(r) e^{-i k \cdot g}.
\]  

(QPBC)

\( \ldots \) \text{if } g \text{ connects the boundaries of a unit cell.)

FB-wave eigenproblem:
Given a wavevector \( k \), look for frequencies \( \omega \in \mathbb{R} \), such that there exist nonzero solutions \((E, H)\) on a unit cell domain, with quasi-periodic boundary conditions (QPBC).

- Outcome:
  \( \exists \omega \text{ with } (E, H) \neq 0: (k, \omega) \in \text{a frequency band, or} \)
  \( \nexists \omega \text{ with } (E, H) \neq 0: \omega \in \text{a bandgap region.} \)

  \( \Rightarrow \) “Bandstructure” calculations.

- QPBC for \( k \) are the same as for \( k + K \), if \( K \cdot g = m 2\pi, m \in \mathbb{Z} \).

  \( \sim \sim \) Restrict \( k \) to the first Brillouin zone.

  (Exclude \( k + K \forall g, m \).)

  (\( K \): A vector of the reciprocal lattice.)

A sequence of dielectric rods

\[ a = 0.4 \text{ \mu m}, \Lambda = 1 \text{ \mu m}, \]

\[ n_b = 1.0, n_g = \sqrt{12}. \]


\begin{itemize}
  \item 1-D periodicity, \( \epsilon(x, z) = \epsilon(x, z + \Lambda) \).
  \item 2-D TE setting, \( E_y(x, z) = ?, (\partial^2_x + \partial^2_z + k^2 \epsilon)E_y = 0. \)  
(*)
  \item Look for FB waves \( E_y(x, z) = u(x, z) e^{-i \beta z} \) \( (\beta \text{ the FB wavenumber, } u(x, z) = u(x, z + \Lambda) \forall z) \)
  \item \( E_y(x, z + \Lambda) = u(x, z + \Lambda) e^{-i \beta(z + \Lambda)} = E_y(x, z) e^{-i \beta \Lambda} \)
  \( \sim \sim \) Restrict (*) to \( z \in [0, \Lambda] \) with boundary conditions \( E_y(x, \Lambda) = e^{-i \beta \Lambda} E_y(x, 0), \partial_z E_y(x, \Lambda) = e^{-i \beta \Lambda} \partial_z E_y(x, 0). \)
  \item Brillouin zone: \( K\Lambda = \pm m 2\pi \sim \sim \beta \in [-\pi/\Lambda, \pi/\Lambda]. \)
\end{itemize}

Defect waveguides

At a frequency in the bandgap of a photonic crystal: \( \exists \) “forbidden” regions \( \sim \sim \) The waves travel elsewhere \( \ldots \)

Line defects in a square lattice of dielectric rods, excitation through conventional waveguides, 2-D QUEP simulations.

- A straight defect waveguide.
- 90° corner in a defect waveguide.
A touch of plasmonics

"Plasmonics": ?

Keywords:
- A branch of photonics.
- Optics involving metals and metal surfaces.
- Interaction between the electromagnetic field and free electrons in the metal / at the surface.
- "Strong" local fields, near field enhancement (nonlinearity).
- "Small" structures: Nan... (Note that, in general, $\eta_m(\omega) > 0$.)

Context of this lecture:
- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Presence of metals: complex (negative) permittivity, strong dispersion, losses; some concepts do not apply.
- Among the phenomena not encountered so far: Surface plasmon polaritons (SPPs).

Surface plasmon polaritons

Surface waves, "plasmon": oscillations of the free electron plasma, "polariton": strong interaction of the optical e.m. field with polarizable matter; here discussed merely as ... (FD)

Optical waves confined at a metal / dielectric interface.

(... accepting the permittivities as given, disregarding any processes in the metal or dielectric that lead to this permittivity.)

Surface plasmon polaritons

Surface plasmon polaritons $\sim \exp(i \omega t)$ (FD)

$\omega c \sqrt{\frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}}$, the dispersion equation for SPPs.

(Note that, in general, $\eta_m(\omega) > 0$.)

Surface plasmon polaritons

$\sim \exp(i \omega t)$ (FD)

$\gamma = \sqrt{\frac{\epsilon_d \epsilon_m}{2}}$, the dispersion equation for SPPs.

(Note that, in general, $\eta_m(\omega) > 0$.)
Surface plasmon polaritons

Characteristic lengths:
- \( x > 0 \): dielectric, \( \epsilon_d = n_d^2 \in \mathbb{R} \).
- \( x < 0 \): metal, \( \epsilon_m \in \mathbb{C} \).

\[
|\phi(x)|^2 \sim e^{-2\kappa_d x} \quad \implies \quad d_d = \frac{1}{2\kappa_d}, \quad \text{(Penetration depth, dielectric.)}
\]

\[
|\phi(x)|^2 \sim e^{2\kappa_m x} \quad \implies \quad d_m = \frac{1}{2\kappa_m}, \quad \text{(Penetration depth, metal.)}
\]

\[
|E|^2 \sim e^{-2\alpha z} \quad \implies \quad L_p = \frac{1}{2\alpha}, \quad \text{the SPP propagation length.}
\]

Field profiles

- SPP, Ag/air, \( \lambda = 0.633 \mu m \), \( \epsilon_m = -14.5 - 1.2i \), \( \epsilon_d = 1.0 \)
- SiO\(_2\)/Si(100 nm)/air, \( \lambda = 0.633 \mu m \), \( \epsilon = 1.45^2 : 3.45^2 : 1.0 \)
- SPP, Ag/air, \( \lambda = 0.633 \mu m \), \( \epsilon_m = -14.5 - 1.2i \), \( \epsilon_d = 1.0 \)
- SiO\(_2\)/Si(100 nm)/air, \( \lambda = 0.633 \mu m \), \( \epsilon = 1.45^2 : 3.45^2 : 1.0 \)
Field profiles

SPP, Ag / air, $\lambda = 1.550 \mu m$

$\epsilon_m = -121 - 4.4i$, $\epsilon_d = 1.0$

$L_p = 812 \mu m$

$\beta/k = 1.0042$

$d_d = 1350 \text{ nm}$

$d_m = 11 \text{ nm}$

Field profiles

SPP, Ag / air, $\lambda = 1.550 \mu m$

$\epsilon_m = -121 - 4.4i$, $\epsilon_d = 1.0$

$L_p = \infty$

$n_{eff} = 1.874$

$d_{air} = 78 \text{ nm}$

Upcoming

Next lectures:

• Oblique semi-guided waves: 2-D integrated optics.
• Summary, concluding remarks.