1. Let S be the set of points (x, y) in $\mathbb{R}^2$ with $-2 \leq x < 3/2,\ 0 < y \leq 4$. Determine all interior points of S, all boundary points of S, the closure of S, and whether S is open, closed, or neither open or closed. Is S connected? Is S bounded? Is S a domain?

2. Assume that Ω is a domain in $\mathbb{R}^2$ bounded by a piecewise smooth closed curve $\partial\Omega$. Consider functions $u, v \in C^2(\bar{\Omega})$.

   (a) Prove *Green's first identity* ($\mathbb{R}^2$)

   $$\oint_{\partial\Omega} u \frac{\partial v}{\partial n}\, \mathrm{d}s = \iint\limits_{\bar{\Omega}} (u\, \Delta v + \boldsymbol{\nabla} u \cdot \boldsymbol{\nabla} v)\ \mathrm{d}A\,. \tag{1}$$

   (b) Suppose that u is harmonic inside the domain, and that u satisfies homogeneous Dirichlet conditions on its boundary:

   $$\Delta u(\boldsymbol{x}) = 0 \ \text{ for } \ \boldsymbol{x} \in \Omega,\ \ u(\boldsymbol{x}) = 0 \ \text{ for } \ \boldsymbol{x} \in \partial\Omega\,. \tag{2}$$

   Show that $u(\boldsymbol{x}) = 0$ for all $\boldsymbol{x} \in \bar{\Omega}$ (Try applying the identity (1) with $u = v$).

3. Consider the following Dirichlet problem in three independent variables

   $$\Delta u(x, y, z) = 0 \ \text{ for } \ 0 < x < a,\ \ 0 < y < b,\ \ 0 < z < c \tag{3}$$

   with homogeneous boundary conditions on five sides of the cuboid

   $$u(0, y, z) = u(a, y, z) = 0,\ \ u(x, 0, z) = u(x, b, z) = 0,\ \ u(x, y, 0) = 0 \tag{4}$$

   and an inhomogeneous Dirichlet condition on the sixth side

   $$u(x, y, c) = f(x, y), \tag{5}$$

   where f is a given smooth function.

   (a) Apply separation of variables to obtain a general Fourier-series solution of the Dirichlet problem.

   (b) Evaluate the solution for the specific inhomogeneity $f(x, y) = 7 \sin(2\pi x/a)\ \sin(5\pi y/b)$.
   Verify explicitly that your solution satisfies the conditions (3), (4), and (5).

   (c) Devise a strategy for solving the Laplace equation (3) subject to the boundary conditions
   $u(0, y, z) = u(a, y, z) = 0,$
   $u(x, 0, z) = 0,\ \ u(x, b, z) = g(x, z),\ \ u(x, y, 0) = 0,\ \ \ u(x, y, c) = f(x, y),$
   where f and g are given smooth functions.

Good luck !